

3.4 – Leslie Model for Population Growth

The following assumptions are made:

- ⇒ Only the female population is considered
- ⇒ Birth rates and survival rates are constant over time
- ⇒ Survival rate is the probability that a female will survive and move into the next age group

Example 1a: A population of bats living in a cave is studied and the following data is collected.

Age (months)	0-6	6-12	12-18	18-24
Initial Population	4500	1800	900	130
Birth Rate	0	1.9	1.5	0.7
Survival Rate	0.5	0.8	0.4	0

To find the number of new bat births after 6 months (1 cycle) multiply the number of female bats in each age group by the corresponding birth rates and find the sum.

The number of female bats who survive in each age group can be found by multiplying the number of female bats in each age group by the corresponding survival rate.

Age (months)	Population	Survival Rate	Number of bats moving to the next age group
0-6	4500	0.5	
6-12	1800	0.8	
12-18	900	0.4	
18-24	130	0	

Find the number of female bats after 12 months (two cycles).

Age (months)	Population	Survival Rate	Number of bats moving to the next age group
0-6		0.5	
6-12		0.8	
12-18		0.4	
18-24		0	

Find the number of female bats after 18 months (three cycles).

Age (months)	Population	Survival Rate	Number of bats moving to the next age group
0-6		0.5	
6-12		0.8	
12-18		0.4	
18-24		0	

Example 2a: A population of kangaroos is studied and the following data is collected.

Age (years)	0-2	2-4	4-6	6-8	8-10
Initial Population	3400	2500	2300	1750	650
Birth Rate	0	0	3.9	2.7	0.9
Survival Rate	0.5	0.8	0.7	0.4	0

Find the number of female kangaroos after 2 years.

Age (years)	Population	Survival Rate	Number of kangaroos moving to the next age group
0-2			
2-4			
4-6			
6-8			
8-10			

Find the number of female kangaroos after 4 years.

Age (years)	Population	Survival Rate	Number of kangaroos moving to the next age group
0-2			
2-4			
4-6			
6-8			
8-10			

Find the number of female kangaroos after 6 years.

Age (years)	Population	Survival Rate	Number of kangaroos moving to the next age group
0-2			
2-4			
4-6			
6-8			
8-10			

Find the number of female kangaroos after 8 years.

Age (years)	Population	Survival Rate	Number of kangaroos moving to the next age group
0-2			
2-4			
4-6			
6-8			
8-10			

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Example 1b: A population of bats living in a cave is studied and the following data is collected.

Age (months)	0-6	6-12	12-18	18-24
Initial Population	4500	1800	900	130
Birth Rate	0	1.9	1.5	0.7
Survival Rate	0.5	0.8	0.4	0

Initial Population matrix:

$$P_0 =$$

Leslie Matrix:

$$L =$$

Find the number of female bats in each age group after 6 months (one cycle).
Then, find the total number of female bats after this cycle.

Find the number of female bats in each age group after 12 months (two cycles).

Find the number of female bats in each age group after three cycles.

Find the number of female bats in each age group after four cycles.

Find the number of female bats in each age group after ten cycles.

Example 2b: A population of kangaroos is studied and the following data is collected.

Age (years)	0-2	2-4	4-6	6-8	8-10
Initial Population	3400	2500	2300	1750	650
Birth Rate	0	0	3.9	2.7	0.9
Survival Rate	0.5	0.8	0.7	0.4	0

Initial Population matrix:

$$P_0 =$$

Leslie Matrix:

$$L =$$

Find the number of female kangaroos in each age group after one cycle.
Then, find the total number of female kangaroos after this cycle.

Find the number of female kangaroos in each age group after two cycles.

Find the number of female kangaroos in each age group after four cycles.

Find the number of female kangaroos in each age group after ten cycles.
Then, find the total number of female kangaroos after this cycle.

After fifteen cycles, how much time has passed?

1a. A population of rats is studied and the following data is collected.

Age (months)	0-3	3-6	6-9	9-12	12-15	15-18
Initial Population	15	9	13	5	0	0
Birth Rate	0	0.3	0.8	0.7	0.4	0
Survival Rate	0.6	0.9	0.9	0.8	0.6	0

Find the total number of female species after 1 cycle.

[illegible]

Find the total number of female species after 2 cycles. How much time is 2 cycles?

[illegible]

Find the total number of female species after 3 cycles. How much time is 3 cycles?

[illegible]

Find the total number of female species after 4 cycles. How much time is 4 cycles?

[illegible]

1b. A population of rats is studied and the following data is collected.

Age (months)	0-3	3-6	6-9	9-12	12-15	15-18
Initial Population	15	9	13	5	0	0
Birth Rate	0	0.3	0.8	0.7	0.4	0
Survival Rate	0.6	0.9	0.9	0.8	0.6	0

Initial Population matrix:

$$P_0 =$$

Leslie Matrix:

$$L =$$

Find the number of female rats in each age group after one cycle.
Then, find the total number of female rats after this cycle.

Find the number of female rats in each age group after two cycles.

Find the number of female rats in each age group after four cycles.

Find the number of female rats in each age group after ten cycles.
Then, find the total number of female rats after this cycle.

After fifteen cycles, how much time has passed?

The rats start dying off early from overcrowding when the total population for a colony reaches 250. Find how long it will take for this to happen.

2a. A population of deer is studied and the following data is collected.

Age (years)	0-2	2-4	4-6	6-8	8-10	10-12
Initial Population	50	30	24	24	12	8
Birth Rate	0	0.8	1.7	1.7	0.8	0.4
Survival Rate	0.6	0.8	0.9	0.9	0.7	0

Find the total number of female species after 1 cycle.

Age	Population	Survival Rate	Number of species moving to the next age group

Find the total number of female species after 2 cycles. How much time is 2 cycles?

[illegible]

Find the total number of female species after 3 cycles. How much time is 3 cycles?

[illegible]

Find the total number of female species after 4 cycles. How much time is 4 cycles?

Age	Population	Survival Rate	Number of species moving to the next age group

2b. A population of deer is studied and the following data is collected.

Age (years)	0-2	2-4	4-6	6-8	8-10	10-12
Initial Population	50	30	24	24	12	8
Birth Rate	0	0.8	1.7	1.7	0.8	0.4
Survival Rate	0.6	0.8	0.9	0.9	0.7	0

Initial Population matrix:

$$P_0 =$$

Leslie Matrix:

$$L =$$

Find the number of female deer in each age group after one cycle.
Then, find the total number of female deer after this cycle.

Find the number of female deer in each age group after two cycles.

Find the number of female deer in each age group after four cycles.

Find the number of female deer in each age group after ten cycles.
Then, find the total number of female deer after this cycle.

After fifteen cycles, how much time has passed?

The deer start dying off early from overcrowding when the total population for a colony reaches 1250. Find how long it will take for this to happen.

3a. A population of buffalo is studied and the following data is collected.

Age (years)	0-1	1-2	2-3	3-4	4-5	5-6
Initial Population	2350	2000	2000	1450	825	400
Birth Rate	0	0	1.5	1.7	1.2	0.4
Survival Rate	0.6	0.7	0.9	0.5	0.3	0

Find the number of female buffalo after 1 year.

Age (years)	Population	Survival Rate	Number of kangaroos moving to the next age group
0-1			
1-2			
2-3			
3-4			
4-5			
5-6			

Find the number of female buffalo after 2 years.

Age (years)	Population	Survival Rate	Number of kangaroos moving to the next age group
0-1			
1-2			
2-3			
3-4			
4-5			
5-6			

Find the number of female buffalo after 3 years.

Age (years)	Population	Survival Rate	Number of kangaroos moving to the next age group
0-1			
1-2			
2-3			
3-4			
4-5			
5-6			

3b. A population of buffalo is studied and the following data is collected.

Age (years)	0-1	1-2	2-3	3-4	4-5	5-6
Initial Population	2350	2000	2000	1450	825	400
Birth Rate	0	0	1.5	1.7	1.2	0.4
Survival Rate	0.6	0.7	0.9	0.5	0.3	0

Initial Population matrix:

$$P_0 =$$

Leslie Matrix:

$$L =$$

Find the number of female buffalo in each age group after one cycle.
Then, find the total number of female buffalo after this cycle.

Find the number of female buffalo in each age group after two cycles.

Find the number of female buffalo in each age group after four cycles.

Find the number of female buffalo in each age group after ten cycles.
Then, find the total number of female buffalo after this cycle.

After fifteen cycles, how much time has passed?

4a. A population of polar bears is studied and the following data is collected.

Age (years)	0-3	3-6	6-9	9-12	12-15
Initial Population	32	54	12	8	1
Birth Rate	0	.9	2.1	1.5	.2
Survival Rate	.8	.6	.5	.2	0

Find the number of females after 1 cycle.

Age	Population	Survival Rate	Number of species moving to the next age group

Find the number of females after 2 cycles.

Age	Population	Survival Rate	Number of species moving to the next age group

Find the number of females after 3 cycles.

Age	Population	Survival Rate	Number of species moving to the next age group

4b. A population of polar bears is studied and the following data is collected.

Age (years)	0-3	3-6	6-9	9-12	12-15
Initial Population	32	54	12	8	1
Birth Rate	0	.9	2.1	1.5	.2
Survival Rate	.8	.6	.5	.2	0

Initial Population matrix:

$$P_0 =$$

Leslie Matrix:

$$L =$$

Find the number of females in each age group after one cycle.
Then, find the total number of females after this cycle.

Find the number of females in each age group after two cycles.

Find the number of females in each age group after four cycles.

Find the number of females in each age group after ten cycles.
Then, find the total number of females after this cycle.

After fifteen cycles, how much time has passed?

5a. A population of lady bugs is studied and the following data is collected.

Age (months)	0-9	9-18	18-27	27-36	36-45
Initial Population	46	81	97	60	53
Birth Rate	0	1.2	1.8	.6	.2
Survival Rate	.4	.7	.5	.9	0

Find the number of females after 1 cycle.

Age	Population	Survival Rate	Number of species moving to the next age group

Find the number of females after 2 cycles.

Age	Population	Survival Rate	Number of species moving to the next age group

Find the number of females after 3 cycles.

Age	Population	Survival Rate	Number of species moving to the next age group

5b. A population of lady bugs is studied and the following data is collected.

Age (months)	0-9	9-18	18-27	27-36	36-45
Initial Population	46	81	97	60	53
Birth Rate	0	1.2	1.8	.6	.2
Survival Rate	.4	.7	.5	.9	0

Initial Population matrix:

$$P_0 =$$

Leslie Matrix:

$$L =$$

Find the number of females in each age group after one cycle.
Then, find the total number of females after this cycle.

Find the number of females in each age group after two cycles.

Find the number of females in each age group after four cycles.

Find the number of females in each age group after ten cycles.
Then, find the total number of females after this cycle.

After fifteen cycles, how much time has passed?

6a. A population of crickets is studied and the following data is collected.

Age (months)	0-4	4-8	8-12	12-16	16-20	20-24
Initial Population	22	22	18	20	7	2
Birth Rate	0	0.5	1.1	0.9	0.4	0
Survival Rate	.6	.8	.9	.8	.6	0

Find the number of females after 1 cycle.

[illegible]

Find the number of females after 2 cycles.

[illegible]

Find the number of females after 3 cycles.

[illegible]

6b. A population of crickets is studied and the following data is collected.

Age (months)	0-4	4-8	8-12	12-16	16-20	20-24
Initial Population	22	22	18	20	7	2
Birth Rate	0	0.5	1.1	0.9	0.4	0
Survival Rate	.6	.8	.9	.8	.6	0

Initial Population matrix:

$$P_0 =$$

Leslie Matrix:

$$L =$$

Find the number of females in each age group after one cycle.
Then, find the total number of females after this cycle.

Find the number of females in each age group after two cycles.

Find the number of females in each age group after four cycles.

Find the number of females in each age group after ten cycles.
Then, find the total number of females after this cycle.

After fifteen cycles, how much time has passed?

7a. A population of wolves is studied and the following data is collected.

Age (months)	0-6	6-12	12-18	18-24
Initial Population	10	0	0	0
Birth Rate	0	3.1	2.1	1.2
Survival Rate	.3	.7	.5	0

Find the number of females after 1 cycle.

Age	Population	Survival Rate	Number of species moving to the next age group

Find the number of females after 2 cycles.

Age	Population	Survival Rate	Number of species moving to the next age group

Find the number of females after 3 cycles.

Age	Population	Survival Rate	Number of species moving to the next age group

7b. A population of wolves is studied and the following data is collected.

Age (months)	0-6	6-12	12-18	18-24
Initial Population	10	0	0	0
Birth Rate	0	3.1	2.1	1.2
Survival Rate	.3	.7	.5	0

Initial Population matrix:

$$P_0 =$$

Leslie Matrix:

$$L =$$

Find the number of females in each age group after one cycle.
Then, find the total number of females after this cycle.

Find the number of females in each age group after two cycles.

Find the number of females in each age group after four cycles.

Find the number of females in each age group after ten cycles.
Then, find the total number of females after this cycle.

After fifteen cycles, how much time has passed?

8a. A population of rabbits is studied and the following data is collected.

Age (years)	0-3	3-6	6-9	9-12	12-15
Initial Population	27	25	21	10	8
Birth Rate	0	.5	1.8	2.1	0.4
Survival Rate	.5	.8	.9	.8	0

Find the number of females after 1 cycle.

Age	Population	Survival Rate	Number of species moving to the next age group

Find the number of females after 2 cycles.

Age	Population	Survival Rate	Number of species moving to the next age group

Find the number of females after 3 cycles.

Age	Population	Survival Rate	Number of species moving to the next age group

8b. A population of rabbits is studied and the following data is collected.

Age (years)	0-3	3-6	6-9	9-12	12-15
Initial Population	27	25	21	10	8
Birth Rate	0	.5	1.8	2.1	0.4
Survival Rate	.5	.8	.9	.8	0

Initial Population matrix:

$$P_0 =$$

Leslie Matrix:

$$L =$$

Find the number of females in each age group after one cycle.
Then, find the total number of females after this cycle.

Find the number of females in each age group after two cycles.

Find the number of females in each age group after four cycles.

Find the number of females in each age group after ten cycles.
Then, find the total number of females after this cycle.

After fifteen cycles, how much time has passed?

If the maximum sustainable population for this animal in its native habitat is 1500 (both males and females), when will the maximum population be reached?

9b. Suppose a beetle population has the characteristics described in the table below:

Age Groups (years)							
Rates	0–5	5–10	10–15	15–20	20–25	25–30	30–35
Birth	0	0.7	1.2	.8	.7	.2	0
Survival	0.5	0.8	0.9	0.9	0.7	0.4	0

The initial female population is given in the table below.

Age Group	0–5	5–10	10–15	15–20	20–25	25–30	30–35
Number	30	30	26	28	32	15	10

Initial Population matrix:

Leslie Matrix:

$$P_0 =$$

$$L =$$

Find the number of females in each age group after one cycle.
Then, find the total number of females after this cycle.

Find the number of females in each age group after two cycles.

Find the number of females in each age group after four cycles.

Find the number of females in each age group after ten cycles.
Then, find the total number of females after this cycle.

After fifteen cycles, how much time has passed?

If the maximum sustainable population for this animal in its native habitat is 12,000 (both males and females), when will the maximum sustainable population be reached?

10a. Suppose that a hamster population has the following birth and survival rates:

Age (years)	Birthrate	Survival Rate
0-2	0	0.7
2-4	0.7	0.8
4-6	1.5	0.9
6-8	1.5	0.8
8-10	0.7	0.6
10-11	0.4	0

The initial population of females is [20 30 25 30 20 10].

Find the number of females after 1 cycle.

[illegible]

Find the number of females after 2 cycles.

[illegible]

Find the number of females after 3 cycles.

[illegible]

10b. Suppose that a hamster population has the following birth and survival rates:

Age (years)	Birthrate	Survival Rate
0–2	0	0.7
2–4	0.7	0.8
4–6	1.5	0.9
6–8	1.5	0.8
8–10	0.7	0.6
10–11	0.4	0

The initial population of females is [20 30 25 30 20 10].

Initial Population matrix:

Leslie Matrix:

$P_0 =$

$L =$

Find the number of females in each age group after one cycle.
Then, find the total number of females after this cycle.

Find the number of females in each age group after two cycles.

Find the number of females in each age group after four cycles.

Find the number of females in each age group after ten cycles.
Then, find the total number of females after this cycle.

After fifteen cycles, how much time has passed?

11. Suppose there is a certain kind of bug that lives at most 3 weeks and reproduces only in the third week of life. Fifty percent of the bugs born in week one will survive into their second week, and seventy percent will survive into their third week. On the average, six new bugs are produced for each bug that survives into its third week. A group of five three week old female bugs decide to make their home in your basement.

a. Fill in the following chart based on the information given about these bugs.

Age (years)	0-1	1-2	2-3
Initial Population			
Birth Rate			
Survival Rate			

b. How long will it be before you have a bug infestation? (The exterminator you consulted said that you needed at least 1000 female bugs before he would treat the situation as an infestation.)

12. A certain species of insect lives four weeks, subject to the following conditions:

- 45% of those in their first week survive to the second week.
- 45% of those in their second week survive to the third week.
- 25% of those in their third week survive to the fourth week.
- On average, 25 new female bugs are produced by each female bug that survives into its fourth week.

a. Fill in the following chart based on the information given about these bugs.

Age (years)	0-1	1-2	2-3	3-4
Initial Population				
Birth Rate				
Survival Rate				

b. If the initial population of females is [700 200 200 100], find the female population distribution after 10 cycles.